Chapter 10
Circles

Section 5
Segment Lengths in Circles

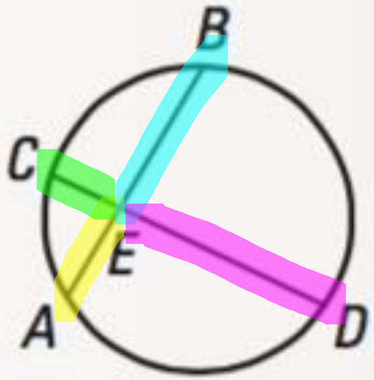
GOAL 1: Finding Lengths of Segments of Chords

When two chords intersect in the interior of a circle, each chord is divided into two segments which are called *segments of a chord*. The following theorem gives a relationship between the lengths of the four segments that are formed.

THEOREM

THEOREM 10.15

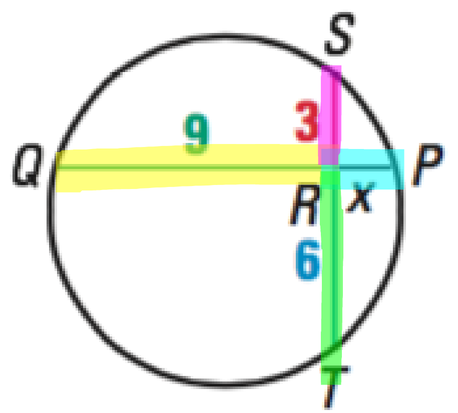
If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.



$EA \cdot EB = EC \cdot ED$

Example 1: Finding Segment Lengths

Chords ST and PQ intersect inside the circle. Find the value of x.



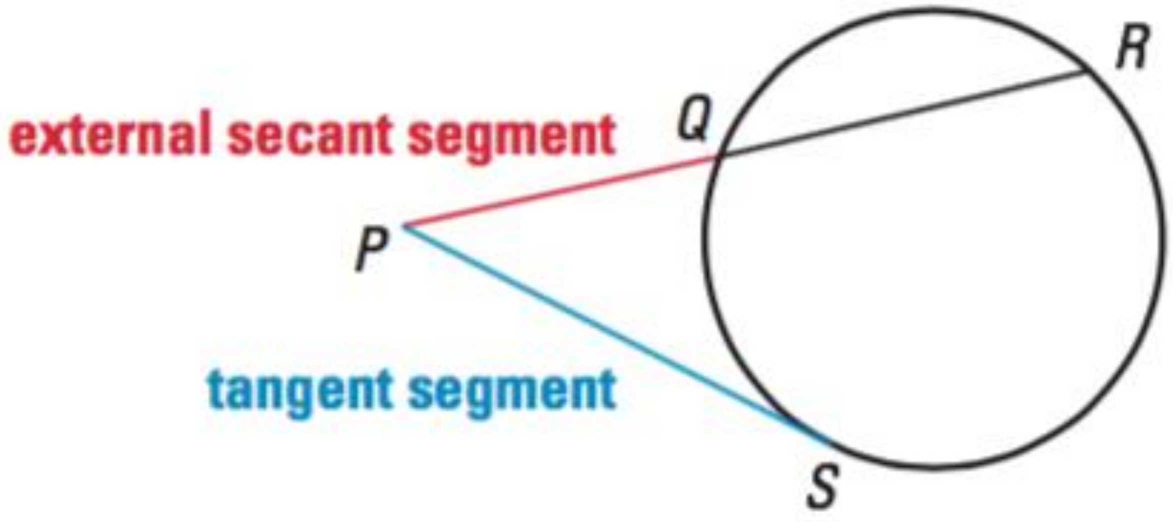
$$9 \cdot x = 6 \cdot 3$$

$$\frac{\cancel{9}x}{\cancel{9}} = \frac{18}{9}$$

$$x = 2$$

GOAL 2: Using Segments of Tangents and Secants

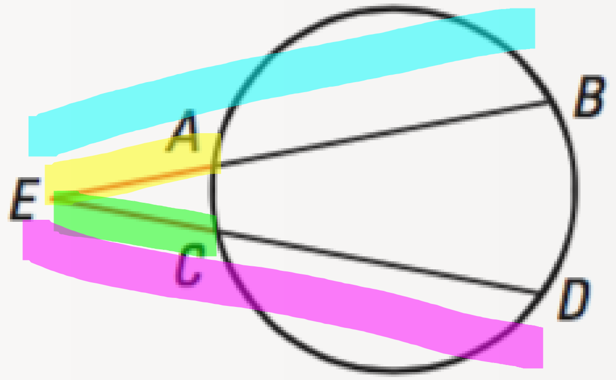
In the figure shown below, $\overline{PS}$ is called a **tangent segment** because it is tangent to the circle at an endpoint. Similarly, $\overline{PR}$ is a **secant segment** and $\overline{PQ}$ is the **external segment** of $\overline{PR}$.



THEOREMS

THEOREM 10.16

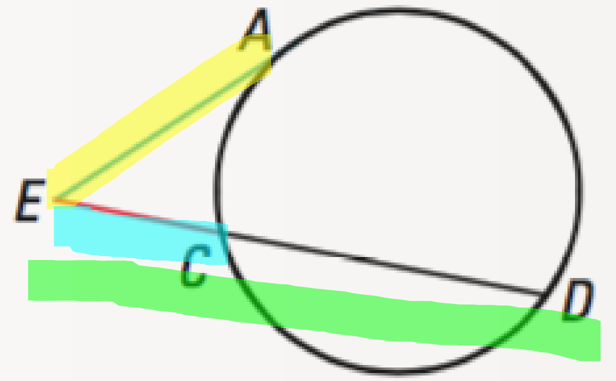
If two secant segments share the same endpoint outside a circle, then the product of the length of one secant segment and the length of its external segment equals the product of the length of the other secant segment and the length of its external segment.



$EA \cdot EB = EC \cdot ED$

THEOREM 10.17

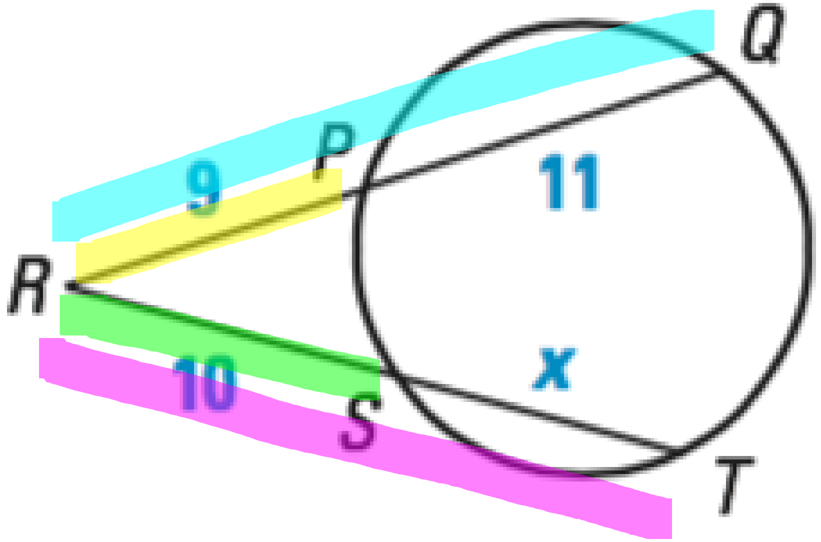
If a secant segment and a tangent segment share an endpoint outside a circle, then the product of the length of the secant segment and the length of its external segment equals the square of the length of the tangent segment.



$(EA)^2 = EC \cdot ED$

Example 2: Finding Segment Lengths

Find the value of x.



$$9 \times 20 = 10(10 + x)$$

$$180 = \cancel{100} + 10x$$

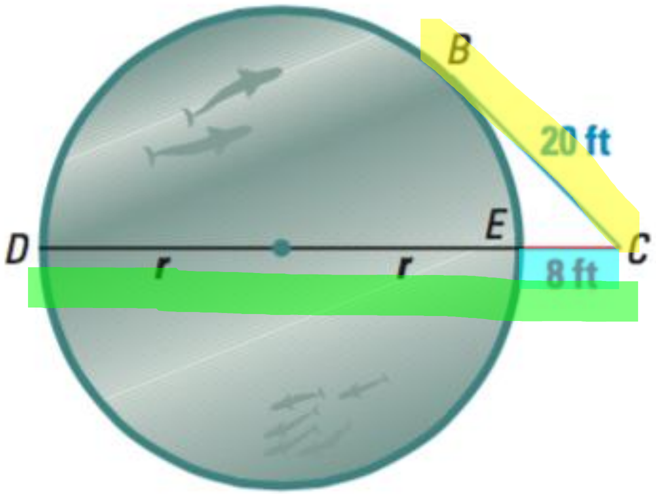
$$-100 \qquad \cancel{-100}$$

$$\frac{80}{10} = \frac{\cancel{10}x}{\cancel{10}}$$

$$8 = x$$

Example 3: Estimating the Radius of a Circle

AQUARIUM TANK You are standing at point *C*, about 8 feet from a circular aquarium tank. The distance from you to a point of tangency on the tank is about 20 feet. Estimate the radius of the tank.



$$20^2 = 8(8+2r)$$

$$400 = \cancel{64} + 16r$$

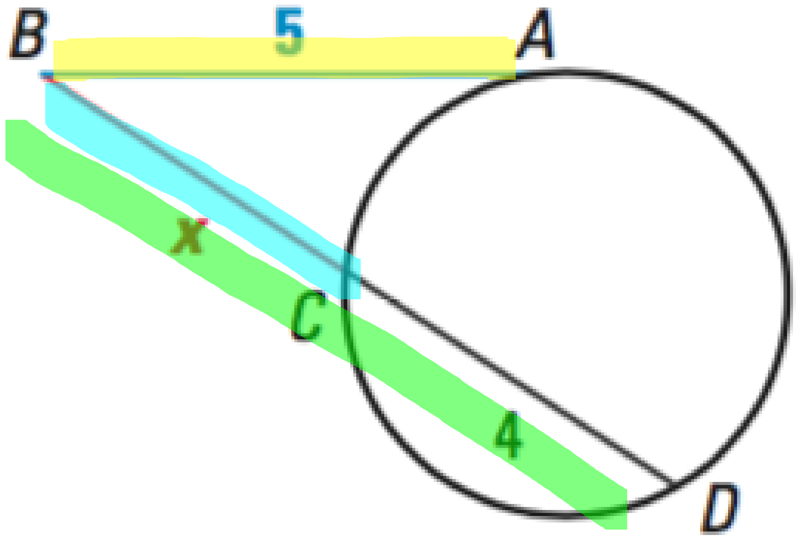
$$-64 \qquad \cancel{-64}$$

$$\frac{336}{16} = \frac{\cancel{16}r}{\cancel{16}}$$

$$21 = r$$

Example 4: Finding Segment Lengths

Use the figure at the right to find the value of x.



$$5^2 = x(x+4)$$

$$\cancel{25} = x^2 + 4x$$

$$-\cancel{25} \qquad -25$$

$$0 = \underset{a}{1}x^2 + \underset{b}{4}x - \underset{c}{25}$$

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\frac{-4 \pm \sqrt{16-4(1)(-25)}}{2(1)}$$

$$\frac{-4 \pm \sqrt{116}}{2} \rightarrow \frac{-4 \pm 10.8}{2}$$

$$\frac{-4+10.8}{2} = \boxed{3.4}$$

$$\frac{-4-10.8}{2} = \cancel{-7.4}$$

EXIT SLIP